COMP 1805 Discrete Structures I
Assignment 2

Due: June 7, 2016 at the end of class

- Write down your name and student number on **every** page. The pages must be **stapled** together.
- You must have a cover page that clearly states **your name, student number, and course number**. If you do not have a cover page with this information, your assignment will not be marked.
- The questions should be answered in order.
- Every part of every question is worth 2 marks. The grading scheme is 2 points for a correct answer, 0 for a completely incorrect answer, and 1 point for something in-between.

1. Determine whether or not the following arguments are valid. If they are valid, then use the rules of inference to show this. If they are invalid, outline precisely why they are invalid.

   (a) If Melanie can program in Java then she can also program in C++. Everyone that can program in C++ has a C++ compiler. Therefore, if Melanie can program in Java, then she has a C++ compiler.

   (b) Everyone who has a computer knows Java. Mike has a computer and Susan knows Java. Therefore, both Susan and Mike have a computer.

   (c) Every student has a laptop. John has a laptop. Tom does not have a laptop. Therefore, Tom is not a student or John is a student.

   (d) Everyone who loves ice cream also loves chocolate. Everyone who loves chocolate also loves Belgium. At least one Carleton student loves ice cream. Therefore, at least one Carleton student loves Belgium.

2. Prove that for every integer n, $3n + 2$ is odd if and only if $9n + 5$ is even.

3. Prove or disprove the following: for any two rational numbers p and q with $p < q$, there is a rational number r such that $p < r < q$.

4. In this question you will prove that $\sqrt{5}$ is irrational. Note: An integer p *divides* an integer q, written $p|q$, if $q = k \times p$ for some integer k. For example, $7|21$ since $21 = 7 \times 3$. If a number does not divide another, we use $\nmid$. For example, $7 \nmid 22$.

   (a) Let n be an integer. Prove that if $5|n^2$ then $5|n$. Hint: consider the contrapositve and consider cases.

   (b) Following the proof that $\sqrt{2}$ is irrational, use the result you have just shown above to prove that $\sqrt{5}$ is also irrational.

5. Let x be a real number such that $x \neq 0$. Prove that $x^2 + 1/x^2 \geq 2$. Hint: Remember that if x is a real number with $x \neq 0$, then x^2 is always strictly greater than 0. Also, try a proof by contradiction here.

6. List the members of the following sets explicitly:

   (a) $\{x \mid x \text{ is a prime number and } 5 < x^2 < 50\}$.

   (b) $\{x \mid x \text{ is the square of an integer and } 0 \leq x < 50\}$.

7. Indicate whether the following propositions are true or false:

   (a) $\{2\} \subset \{1, 2, 5\}$
   (b) $\{\{1\}, \{1, 2\}, \{3\}\} \subseteq \{\{1\}, \{2\}, \{3\}, \{1, 2\}\}$
   (c) $\{1\} \notin \{1, 2, 3\}$
   (d) $2 \in \{\{1\}, \{2\}, 3\}$
   (e) $\{\{2\}\} \subseteq \{\{1\}, \{2\}, 3\}$

8. Let A, B and C be sets. Determine whether or not the following equivalences are valid. If they are valid, show this using set identities. Otherwise, give a counter-example: three sets A, B, and C, along with the universe U, for which the two expressions are different.

   (a) $(A \cap (B - C)) = ((A - C) \cap B)$
   (b) $\overline{(A - B) \cup (B - A)} = (A \cap B)$
   (c) $(A - \overline{B}) \cup (\overline{B} - A) = (A \cup \overline{B}) \cap (B \cup \overline{A})$

9. Draw the Venn Diagram of the following:

   (a) $(A \cup B) \cap C$
   (b) $(A \cap B) - C$
   (c) $\overline{(A \cup C)} \cap B$

10. Determine whether or not the following functions from real numbers to real numbers are bijections. If they are bijections, then find the inverse. If they are not bijections, then explain why not.

    (a) $f(x) = -5x$
    (b) $f(x) = |2x - 6|$
    (c) $f(x) = 2x^2 - 6x + 3$
    (d) $f(x) = x^3 - 9$

11. Let $A = \{1, 2, 3, 4\}$, $B = \{$“don’t”, “know”, “how”, “to”$\}$, and $C = \{$“count”, “no”, “more”$\}$. Using these sets as domain and co-domain, define functions having the following properties and explain why the properties hold:

    (a) A function that is not injective and not surjective.
    (b) A function that is surjective, but not injective.
    (c) A function that is injective, but not surjective.
    (d) A function that is bijective.

12. Let f and g be functions from the real numbers to the real numbers, with $f(x) = 2x - 15$ and $g(x) = x^2 - 12$. Give the following functions (simplify as much as possible):

    (a) $f \circ g$
    (b) $g \circ f$
    (c) $(f \circ g) \circ f$