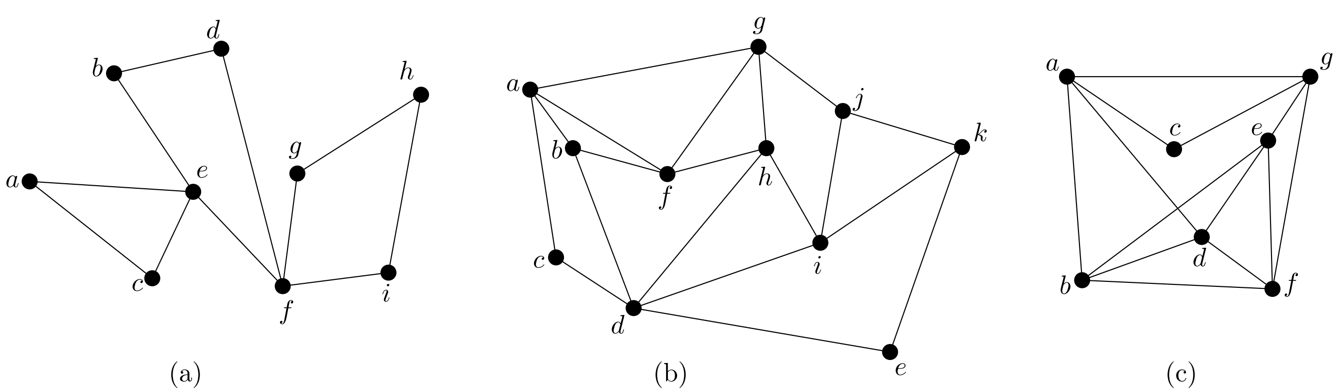
COMP 1805 Discrete Structures I
Assignment 5

Due: August 16th 2016, at the end of class

- Write down your name and student number on **every** page. The pages must be **stapled** together.
- You must have a cover page that clearly states **your name, student number, and course number**. If you do not have a cover page with this information, your assignment will not be marked.
- The questions should be answered in order.
- Every part of every question is worth 2 marks. The grading scheme is 2 points for a correct answer, 0 for a completely incorrect answer, and 1 point for something in-between.

1. Recall that a graph is *bipartite* if and only if it is possible to colour the vertices with two colours, such that any two adjacent vertices have different colours. In this question, we will prove that all trees are bipartite by induction on the number of vertices.
   (a) Prove that a tree with one vertex is bipartite.
   (b) State the inductive hypothesis.
   (c) State what needs to be shown for the inductive step.
   (d) Prove the inductive step. You may use the fact that every tree with at least two vertices has a leaf (a vertex of degree 1).
2. Draw the following graphs.
   (a) K_8
   (b) C_7
   (c) $K_{6,5}$
3. For each of the graphs in Figure 1, determine whether they have (i) an Euler cycle, and/or (ii) a Hamilton cycle. If the cycle exists, list the order in which it visits the vertices. Otherwise, just say 'No'.



(a) (b) (c)

Figure 1: The graphs for Questions 3 through 5.

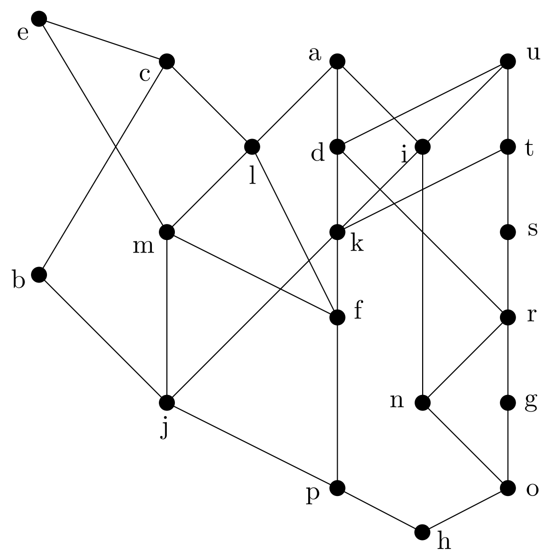
4. For each of the graphs in Figure 1, list the vertices in the order a depth-first search starting from vertex a would visit them. If there are multiple choices, choose the vertex earlier in the alphabet.

5. For each of the graphs in Figure 1, list the vertices in the order a breadth-first search starting from vertex a would visit them. If there are multiple choices, choose the vertex earlier in the alphabet.

6. Given the set $A = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$, determine whether the following relations on A are reflexive, symmetric, anti-symmetric, transitive, equivalence relations, and/or partial orders. If it is a partial order, draw the Hasse diagram.

   (a) $R_1 = \{(a, b) | a \leq b\}$.
   (b) $R_2 = \{(a, b) | a^3 = b^3\}$.
   (c) $R_3 = \{(a, b) | ab \leq 0\}$.
   (d) $R_4 = \{(a, b) | a > b^2\}$.

7. Consider the Hasse diagram below. Use topological sort to compute a valid linear order of the elements.



8. Let A be the set of all bit strings of length 25. Let R be the relation defined on A where two bit strings are related if they have the same number of ones.

   (a) Show that R is an equivalence relation.
   (b) Enumerate one bit string from five different equivalence classes of R.